IMPROVED LARGE APERTURE WAVEGUIDE COUPLING THEORY

Ralph Levy
Microwave Development Laboratories , Inc.
11 Michigan Drive, Natick, Mass.01760 USA

ABSTRACT

Bethe's small aperture coupling theory, modified by Cohn for large coupling apertures,is improved by including correction terms obtained by averaging the fields over the large aperture. Additionally , inclusion of non-empirical thickness correction factors derived previously by McDonald give coupling formulas which result in theoretical predictions for multi-aperture couplers substantially in exact agreement with experiment (correcting small discrepancies previously noted by the author in a 1968 paper).

Introduction

The theory of microwave coupling by large apertures has developed in a number of stages,originating in Bethe's small aperture coupling theory of 1943[1,2]. A major extension of Bethe's work was described by Cohn in 1952 [3],and enabled the theory to be applied to large apertures of finite thickness. Cohn recognized that a coupling aperture between two waveguides has an equivalent circuit representation involving lossless impedances,which must therefore obey Foster's reactance theorem. Hence to take account of the aperture resonance, the impedance was modified simply by inclusion of a factor $(1-f^2/f_0^2)$, where f is frequency and f_0 the resonant frequency of the aperture. The effect of finite thickness was taken into account by treating the aperture as a finite length of waveguide beyond cut-off. However it was noted that this thickness correction was somewhat empirical, and "effective thickness" factors had to be included to give reasonable agreement between theory and experiment.

The Bethe/Cohn theory was applied to the analysis and synthesis of multi-aperture waveguide directional couplers by the author in 1968 [4]. It was shown to give excellent results for prediction of the directivity of multi-aperture couplers, and the coupling could be predicted to within 0.3 dB over most of a complete waveguide band. On the other hand at high frequencies, between f/f_c values of 1.6 and 1.8 , the discrepancy in coupling increased gradually from 0.3 dB to 0.7 dB, independently of the absolute coupling value or the number of coupling apertures. A second unsatisfactory feature was the inclusion of the empirical correction factor for finite coupling wall thickness, a factor which was obtained by matching the theory to the experimental results to a considerable extent. Hence one could justifiably question the validity of the original theoretical basic,even though the design technique was superior to previous methods.

Fortunately the thickness correction factor has now been obtained rigorously in a paper by McDonald[5] and it will be shown that inclusion of his thickness correction factors in a modified Bethe-Cohn aperture coupling theory gives precise results , requiring no empirical adjustments.

However in the interim a number of papers by Pandharipande and Das have (or will) be published , e.g.[6] , giving excellent agreement between theory and experiment for single apertures in the form of long rectangular slots. In this work the aperture resonance factor of Cohn [3] appears naturally as a direct consequence of the e.m. theoretical approach adopted, and there is no recourse to measured polarizabilities for slot apertures as used in all previous work. The disadvantage of this approach is that complicated e.m. theory must be worked out for each aperture shape,but perhaps more importantly the theory does not build upon previous work. Thus no results are as yet given for circular or elliptical shaped apertures where Bethe[2] has given exact expressions for aperture polarizabilities. If a modification to Bethe's theory could be found to correct the frequency dependence of the coupling, and if this could be as simple in form as that developed by Cohn [3] for finite aperture size , then there would be no need to discard the work of the past 35 years in favor of a new and more complex method.

Modified Bethe-Cohn aperture coupling theory

A basic premise of the original Bethe theory was the assumption of an aperture small compared to a wavelength, so that the field could be considered as uniform over the aperture. The introduction of the aperture resonance term by Cohn results in a great improvement by guaranteeing correct results in the limiting cases of small apertures and resonant apertures. There is a significant improvement for intermediate cases also , but there seems little reason to assume that the theory will be precise for such cases.

Now Bethe's theory assumes that the coupling is expressed in terms of electric and magnetic dipoles excited by fields which would be present at the aperture if it were not there. A natural extension is surely to average the field over the cross section of the aperture, resulting in a further correction term in addition to that derived by Cohn. This averaging process will not affect the limiting cases of infinitesimally small and resonant apertures.

As an example of the field averaging technique consider the cases of transverse and longitudinal slots in the common broad wall of two identical coupled waveguides, as indicated in Fig.1.

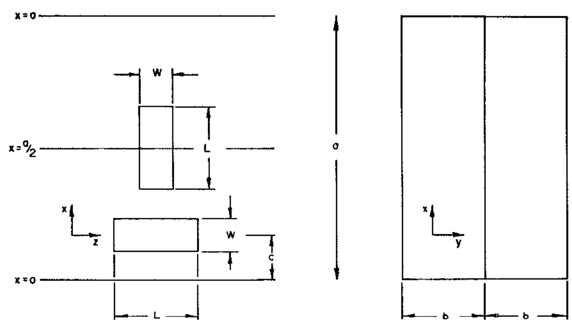

Fig.1 Longitudinal and transverse slots coupling common broad wall of two waveguides (slots need not be identical)

The field components are proportional to the following expressions:

$$H_x = -\sin\frac{\pi x}{a}\, e^{-j2\pi z/\lambda_g}$$

$$H_z = j\frac{\lambda_g}{2a}\cos\frac{\pi x}{a}\, e^{-j2\pi z/\lambda_g}$$

$$E_y = \frac{\lambda_g}{\lambda}\sin\frac{\pi x}{a}\, e^{-j2\pi z/\lambda_g} \qquad (1)$$

In the case of the centered transverse slot of Fig.1, the coupling is mainly via the H_x field if the slot width w is narrow. The modified Bethe coupling theory now requires an average value of H_x^2 , and the exponential term may be ignored in this case. The required correction factor for H_x^2 is therefore

$$A(H_x^2) = \int_{(a-L)/2}^{(a+L)/2} \sin^2\frac{\pi x}{a}\,dx \Big/ \int_{(a-L)/2}^{(a+L)/2} dx$$

$$= \frac{1}{2}\left(1 + \frac{\sin\frac{\pi L}{a}}{\frac{\pi L}{a}}\right) \qquad (2)$$

The case of the narrow longitudinal slot is somewhat different,since the main coupling is via the H_z field component. Conventionally the exponential term is ignored,but in the averaging procedure it is necessary to consider the instantaneous sinusoidal variation of the field over the aperture in the direction of propagation, z. Hence the z-variation must be taken as $\cos\frac{2\pi z}{\lambda_g}$, with z=o at the centre of the slot. Finally the averaging factor for H_z^2 is given by

$$A(H_z^2) = \int_{-L/2}^{L/2} \cos^2\frac{2\pi z}{\lambda_g}\,dz \Big/ \int_{-L/2}^{L/2} dz$$

$$= \frac{1}{2}\left(1 + \frac{\sin\frac{2\pi L}{\lambda_g}}{\frac{2\pi L}{\lambda_g}}\right) \qquad (3)$$

A significant difference between A(H_x^2) and A(H_x^2) now appears, since A(H_x^2) represents a small frequency-independent attenuation, whereas A(H_z^2) represents frequency-dependent attenuation.

In order to use these terms, the standard (Cohn) correction factor, e.g. [4, Equ.(12)] is simply multiplied by the appropriate term. These are always <1 , so that weaker coupling is predicted compared with the unmodified Bethe-Cohn theory. In the case of apertures having finite dimensions in the z direction, an additional frequency dependence occurs. This results in looser coupling at higher frequencies, as required experimentally.

In cases where the apertures have more complicated shapes than the simple rectangles of Fig.1, e.g.round-ended slots, it may be necessary to carry out the integration numerically, but this is easily incorporated in a computer program. Since practical coupling slots are often quite wide it is also desirable to take all three field components into account, and if incorporated within the computer program this scarcely complicates practical implementations.

The more general averaging is illustrated by the popular circular coupling aperture shown in Fig.2.

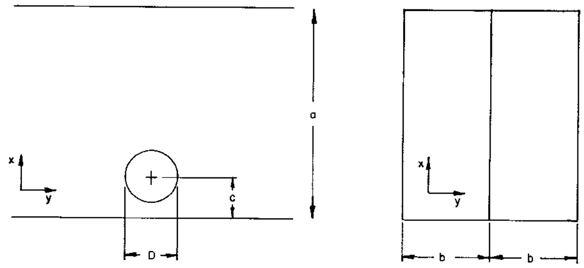

Fig.2 Circular coupling hole

Here the averaging factor for the H_x field is given by

$$A(H_x^2) = \iint \sin^2\frac{\pi x}{a}\cos^2\frac{2\pi z}{\lambda_g}\,dxdz \Big/ \iint \sin^2\frac{\pi c}{a}\,dxdz$$

$$= \iint \sin^2\frac{\pi x}{a}\cos^2\frac{2\pi z}{\lambda_g}\,dxdz \Big/ \left(\frac{\pi}{4}D^2\sin^2\frac{\pi c}{a}\right) \qquad (4)$$

where the double integrations are performed over the surface of the aperture. Similar expressions may be stated for the H_z and E_y field components.

Thickness correction factor

Initial tests of the new theory were performed using the original semi-empirical correction factor , which in the case of circular holes is effected by replacing the actual coupling wall thickness t by At, where

$$A = 1 + \alpha\, D/t \qquad (5)$$

D is the hole diameter , and α is a constant , given originally as 0.065 in [4, Equ.(15)]. The results showed that the new theory predicted the shape of the coupling(i.e.the detailed frequency-dependence) very accurately, but α needed to be reduced to ~0.03 to predict the correct absolute coupling value.

A formal electromagnetic theory of the coupling through thick apertures had been available for several years [5], but it was difficult to apply while the theoretical coupling curves were displaced from practice. When this situation was corrected using the new field-averaging correction factors, it was soon established that McDonald's theory gave results in very close agreement with the empirical equation (5).

McDonald's theory is applicable only to small apertures,and provides a precise correction factor for thickness. It is necessary still to include both Cohn's correction factor for aperture resonance and the field-averaging factors introduced here. McDonald presents thickness correction factors for both circular holes and rectangular slots, and the results may be computerized in a few lines using curve-fitting techniques.

Practical Results

The new field-averaging correction factors combined with McDonald's thickness correction factor give theoretical results which are in exact agreement with experiment as far as can be determined(i.e. within experimental error). This statement has been proven for a majority of the cases examined in all waveguide sizes, for coupling values ranging from 3dB through 50 dB, for apertures consisting of circular holes or round-ended slots, and for any number of apertures. (Some discrepancies discovered in old experimental results are thought to represent experimental errors).

A typical example is the 6-aperture WR-430 12.5 dB broad-wall coupler given previously in 1968,[4,Fig.12]. This is reproduced in Fig.3

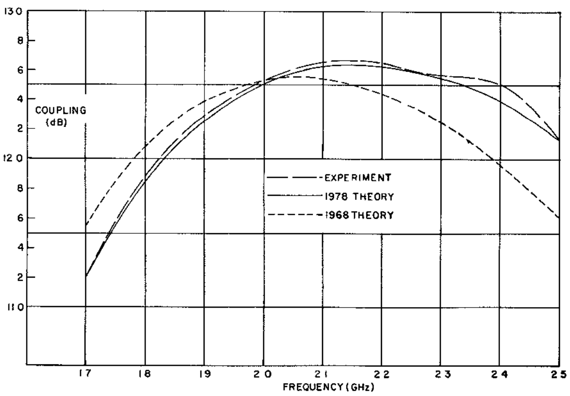

Fig.3 WR430 6-aperture coupler(c.f.[4,Fig.12]

with the addition of the latest theory, and shows agreement for coupling to within 0.1 dB across the entire waveguide band. Results in smaller waveguides have been almost as good, with any errors in coupling as large as ±0.4 dB being correctable by making a maximum change in coupling wall thickness of ±8%, i.e. ±0.003" for a .040" wall. Such discrepancies can be partially ascribed either to burrs or rounded edges of the apertures. Sensitivity analysis shows that a tolerance of ±0.001 in. on slot or hole dimensions of the order of 0.4 in.may cause an error of approximately ±0.1 dB in coupling.

A second example illustrating the use of transverse and longitudinal slots [7] is shown in Fig.4,

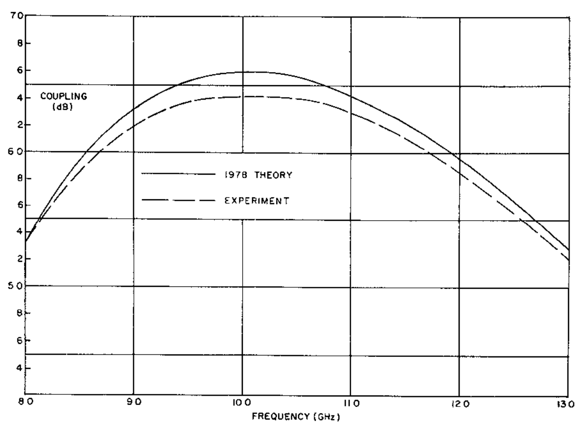

Fig.4 WR90 15-Aperture 6-dB coupler using slots

and here the agreement between theory and experiment is better than 0.2 dB.

References

[1] H.A.Bethe,"Theory of Side Windows in Waveguides", M.I.T. Rad.Lab.Report No.43-27, April 1943.

[2] H.A.Bethe,"Theory of Diffraction by Small Holes", Phys.Rev. Vol.66, pp. 163-182, October 1944.

[3] S.B.Cohn,"Microwave Coupling by Large Apertures", Proc. IRE, Vol.40, pp.696-699, June 1952.

[4] R.Levy,"Analysis and Synthesis of Waveguide Multi-aperture Directional Couplers", IEEE Trans. on Microwave Theory and Techniques,Vol.MTT-16, pp.995-1006, December 1968.

[5] N.A.McDonald,"Electric and Magnetic Coupling through Small Apertures in Shield Walls of any Thickness",IEEE Trans.on Microwave Theory and Techniques, Vol. MTT-20, pp.689-695,October 1972.

[6] V.M.Pandharipande and B.N.Das,"Coupling of waveguides through Large Apertures", IEEE Trans. on Microwave Theory and Techniques,Vol.MTT-26, pp. 209-212, March 1978.

[7] H.J.Riblet and T.S.Saad,"A New Type of Waveguide Directional Coupler", Proc.IRE, Vol.36, pp.61-64, January 1948.